Eur. Phys. J. C 29, 447–451 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01233-2

THE EUROPEAN
PHYSICAL JOURNAL C

Symplectic quantization of open strings in constant background B-field

J. Jing[1,2,a], Z.-W. Long[3,b], L.-J. Tian[2,c], S. Jin[2,d]

[1] Department of Physics and Electronic, School of Science, Beijing University of Chemical Technology, Beijing 100029, P.R. China
[2] Theoretical Physics Division, NanKai Institute of Mathematics, NanKai University, Tianjin 300071, P.R. China
[3] Department of Physics, GuiZhou University, GuiYang 550025, P.R. China

Received: 3 April 2003 / Revised version: 18 April 2003 /
Published online: 18 June 2003 –

Abstract. The symplectic quantization (Faddeev–Jackiw) method is reviewed briefly, and then it is applied to the open strings in the D-brane background with a non-vanishing constant B-field. We shall work in the discrete version, and the reduced phase space is obtained directly by solving the mixed boundary conditions. The non-commutativity of coordinates along the D-brane is reproduced. Some ambiguities in the previous papers could be avoided by this method.

1 Introduction

The concept of non-commutativity has a long history in physics [1], and it has attracted much attention in the past few years [2] owing to the inspiration of superstring theories. Nowadays, it is widely believed that the open strings attached to D-branes in the presence of a background B-field would induce non-commutativity in its end points, i.e. along the D-brane's world volume [3–5]. The most conventional way to derive this non-commutativity is to employ Dirac brackets [6], which were proposed by Dirac more than half a century ago, and treat the mixed boundary conditions (BCs) as primary constraints. However, such primary constraints have a different origin compared to that of the traditional Dirac's context in which the primary constraints were introduced due to the singularity of the Lagrangian, so a proper treatment of the BCs is needed.

Recently, there were some renewed discussions on this subject [7,8], and some discrepancies appeared. One of the focuses of these ambiguities is how to treat the mixed BCs, as mentioned above; the BCs are not the primary constraints in the standard Dirac's context. In the [3,5,9], these BCs were treated as primary Dirac constraints; subsequently, an infinite set of secondary second class constraints could be obtained by the consistency requirements. This is hard to understand, because the end points would live in a negative infinite-dimensional phase space, and it is quite amazing that such circumstances rarely happened before. In a recent paper [7], the author announced that if the BCs were treated as primary constraints, then the Dirac method would not lead to an infinite set of secondary constraint chains but to a finite one; also, the non-commutative algebras would not appear. So it is necessary to discuss this problem in a different way.

Some attempts were made to avoid the discussion of the constraints in [8,10]; there, the authors try to modify the symplectic structure, i.e, the Poisson brackets, to avoid such problems. In [8] the authors find that there are infinite possible results, and neither of them is superior to the others.

In this paper, we shall analyze this problem in an alternative way; that is, we shall apply the so-called symplectic quantization method (which was proposed by Faddeev and Jackiw [11], therefore the FJ method for short) to this problem [12]. The advantage of this method is that one does not need know all the constraint chains by the consistency requirements, and in the classification of the constraints into the so-called primary or secondary ones, the first class or the second class is not needed also, so the ambiguities mentioned above could be avoided.

The organization of this paper is as follows. In Sect. 2, we shall review the FJ method briefly; Sect. 3 is devoted to an analysis of the open strings in constant background B-field by using the FJ method, and finally, some conclusions and discussions will be given in the last section.

[a] e-mail: Jian.Jing@eyou.com
[b] e-mail: longshc@hotmail.com
[c] e-mail: tianlijun96@eyou.com
[d] e-mail: jinshuo@eyou.com

2 A brief review of the Faddeev–Jackiw method

The Faddeev–Jackiw method [11] is an alternative to the Dirac procedure to deal with constrained systems. Compared to Dirac's method, it can avoid the classification of the constraints into the primary and secondary ones, and first and second class ones, and gives us a simple way to calculate the commutators. In this section, we shall outline this method briefly, and for the sake of simplicity, we only focus on the systems which have a finite number of degrees of freedom N; the generalization to the field theories is straightforward.

The starting point of the FJ method is the first-order Lagrangian,

$$L(\xi, \dot{\xi}) = a_i(\xi)\dot{\xi}^i - H(\xi), \quad i = 1, \cdots, 2N. \tag{1}$$

The variables ξ^i are defined as follows:

$$\begin{aligned} \xi^i &= q^i, \quad i = 1, 2, \cdots, N, \\ \xi^{N+1} &= p_i. \end{aligned} \tag{2}$$

This first-order form Lagrangian is quite general, and the Lagrangians which are of higher order in the time derivatives can be rewritten in this form by introducing new canonical variables. The Euler–Lagrange equations can be obtained by the variation of (1):

$$\frac{\partial L}{\partial \xi^i} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\xi}^i} = 0, \tag{3}$$

or, equivalently, by substituting (1) to the Euler–Lagrange equation (3), it can also be written as

$$f_{ij}\dot{\xi}^j = \frac{\partial}{\partial \xi^i} H, \tag{4}$$

where $f_{ij} = \frac{\partial}{\partial \xi_i} a_j - \frac{\partial}{\partial \xi_j} a_i$ is a $2N \times 2N$ matrix.

In the case of the matrix f_{ij} not being degenerate, i.e, f^{ij}, the inverse of f_{ij}, exists, the ξ^i satisfy the evolution equations

$$\dot{\xi}^i = f^{ij}\frac{\partial}{\partial \xi^j} H. \tag{5}$$

On the other hand, the evolution of the variables is determined by the Hamiltonian

$$\dot{\xi}^i = \{\xi^i, H\} = \{\xi^i, \xi^j\}\frac{\partial H}{\partial \xi^j}, \tag{6}$$

and comparing (5) and (6), the conclusion that the commutation relations among the variables are given by f^{ij}, i.e, the inverse of f_{ij}, can be drawn. The results can be further simplified by the Darboux theorem. According to the Darboux theorem, we can construct a coordinate transformation $\xi^i \to Q^i(\xi)$, so that the canonical one-form $a_i(\xi)\mathrm{d}\xi^i$ in the Lagrangian (1) acquires the diagonal form

$$a_i(\xi)\mathrm{d}\xi^i = \frac{1}{2}Q^i(\xi)\omega_{ij}\mathrm{d}Q^j(\xi), \tag{7}$$

where

$$\omega_{ij} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}_{ij}. \tag{8}$$

The price for doing so is that the Hamiltonian in the new variables Q^i will be more complex generally.

In the case that the matrix f_{ij} is degenerate, the Lagrangian (1) is singular, which means that it describes a constrained system. The Darboux theorem can still be applied for the maximal $2n \times 2n$ (of course $n \le N$) nondegenerate subblock of f_{ij}, and the Lagrangian (1) transformed as follows:

$$\begin{aligned} L(Q^i, \dot{Q}^i, z) = \frac{1}{2}Q^i\omega_{ij}\dot{Q}^j - \Phi(Q^i, z), \\ i, j = 1, \cdots, 2n, \end{aligned} \tag{9}$$

where the z_i are the $2N - 2n$ coordinates which are left unchanged. Then, we apply the Euler–Lagrange equation to the variables z_i,

$$\frac{\partial \Phi}{\partial z^i} = 0, \tag{10}$$

to solve as many z_i as possible in terms of the Q^i. However, if the matrix $\frac{\partial^2 \Phi}{\partial z^i z^j}$ is singular, then we cannot solve all the z_i. Eliminating as many z_i type variables as possible, we reach an expression which depends on the left z type variables linearly. After this step is completed, the Lagrangian (9) can be written as

$$L(Q^i, \dot{Q}^i, z) = \frac{1}{2}Q^i\omega_{ij}\dot{Q}^j - H(Q^i) - \lambda_i\Psi^i(Q), \tag{11}$$

where we have renamed the remaining variables z_i as λ_i. From the above equation, we see that in fact the λ_i are the Lagrange multipliers and the $\Psi^i(Q)$ are the real constraints, $\Psi^i(Q) = 0$. Elimination should go on further by solving the constraints $\Psi^i(Q) = 0$ and then substituting them in (11). A new Lagrangian $L(\eta) = b_i(\eta)\dot{\eta}^i - W(\eta)$ will be gotten with a smaller number of variables. Then the whole procedure should be repeated until a unconstrained space, i.e, the reduced phase space and a Lagrangian like (1) are obtained. Finally, the commutators among this set of new variables can be read from the inverse of the matrix $f'_{ij} = \frac{\partial b_j}{\partial \eta^i} - \frac{\partial b_i}{\partial \eta^j}$.

3 The model

The action for an open string with its end points attached on a D-brane in the presence of an NS B-field is (our conventions are almost the same as [5])

$$\begin{aligned} S = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma[&g^{\alpha\beta}\eta_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu \\ &+2\pi\alpha' B_{\mu\nu}\epsilon^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu] \\ &+\int \mathrm{d}\tau A_\mu\partial_\tau X^\mu|_{\sigma=\pi} - \int \mathrm{d}\tau A_\mu\partial_\tau X^\mu|_{\sigma=0}, \end{aligned} \tag{12}$$

where $g_{\alpha\beta} = \mathrm{diag}(-,+)$, $\epsilon^{01} = -\epsilon^{10} = 1$, $B_{\mu\nu} = -B_{\nu\mu}$, $\eta_{\mu\nu} = \mathrm{diag}(-,+,\cdots,+)$, and the length of the string is

π. In the case of both end points attached on the same D-brane, the last two terms can be written as

$$-\frac{1}{2\pi\alpha'}\int \mathrm{d}^2\sigma F_{\mu\nu}\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu$$

and the action (12) is

$$S=\frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma[g^{\alpha\beta}\eta_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu +2\pi\alpha'\mathcal{F}_{\mu\nu}\epsilon^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu], \quad (13)$$

where $\mathcal{F}=B-F=B-\mathrm{d}A$, which is invariant under both a $U(1)$ gauge transformation, $A\to A+\mathrm{d}\lambda$, and the so-called Λ translation, $A\to A+\Lambda, B\to B+\mathrm{d}\Lambda$. Without loss of any generality, we put the electric mixing $\mathcal{F}_{0\mu}=\mathcal{F}_{\mu 0}=0$ [5], and for the sake of simplicity, we set $2\pi\alpha'=1$ and recover it when it is necessary.

The canonical momenta conjugate to the X^μ are

$$P_\mu=\frac{\delta S}{\delta \dot{X}^\mu}=-\partial_\tau X_\mu+\mathcal{F}_{\mu\nu}\partial_\sigma X^\nu. \quad (14)$$

The variation of (13) gives both the equation of motion and the mixed BCs, respectively,

$$(\partial_\tau^2-\partial_\sigma^2)X^\mu=0, \quad (15)$$

$$(\partial_\sigma X^\mu-\mathcal{F}^\mu{}_\nu\partial_\tau X^\nu)_{\sigma=0,\pi}=0. \quad (16)$$

The BCs (16) can be written in terms of canonical variables X^μ and P_μ:

$$(M_{\mu\nu}\partial_\sigma X^\nu+\mathcal{F}_\mu{}^\nu P_\nu)_{\sigma=0,\pi}=0. \quad (17)$$

From the above BCs, we can see that it is inconsistent to impose the Poisson brackets as usual,

$$\begin{aligned}\{X^\mu(\tau,\sigma),\quad P_\nu(\tau,\sigma')\}&=\delta^\mu_\nu\delta(\sigma-\sigma'),\\ \{X^\mu(\tau,\sigma),\quad X^\nu(\tau,\sigma')\}&=0,\\ \{P_\mu(\tau,\sigma),\quad P_\nu(\tau,\sigma')\}&=0. \end{aligned}\quad (18)$$

So a careful treatment of these BCs is needed. In [5,9,3], the above BCs are treated as primary constraints, and the Dirac methods were employed to derive the non-commutative algebras. Because these BCs are only valid on the D-brane's world volume, some singularities just as $\delta(\sigma)$ or $\delta(\sigma-\pi)$ (or even the derivatives of these terms) must be introduced [4]. It is also a tedious task to find all the constraints and then calculate the Dirac brackets. In order to avoid such singularities and the calculation of Dirac brackets, we shall work in the discrete version, which means that we discretize σ, and denote the steps by $\varepsilon=\frac{\pi}{N}$, so that the continuum theory can be obtained by taking the limit $\varepsilon\to 0$ or $N\to\infty$.

The action and the BCs in the discrete version are

$$\begin{aligned}S=\frac{1}{2}\int \mathrm{d}t\Big[&-\varepsilon\eta_{\mu\nu}\dot{X}_0^\mu\dot{X}_0^\nu-\varepsilon\eta_{\mu\nu}\dot{X}_1^\mu\dot{X}_1^\nu-\cdots\\ &-\varepsilon\eta_{\mu\nu}\dot{X}_{N-1}^\mu\dot{X}_{N-1}^\nu-\varepsilon\eta_{\mu\nu}\dot{X}_N^\mu\dot{X}_N^\nu\\ &+\frac{1}{\varepsilon}\eta_{\mu\nu}(X_1-X_0)^\mu(X_1-X_0)^\nu\\ &+\frac{1}{\varepsilon}\eta_{\mu\nu}(X_2-X_1)^\mu(X_2-X_1)^\nu+\cdots\\ &+\frac{1}{\varepsilon}\eta_{\mu\nu}(X_{N-1}-X_{N-2})^\mu(X_{N-1}-X_{N-2})^\nu\\ &+\frac{1}{\varepsilon}\eta_{\mu\nu}(X_N-X_{N-1})^\mu(X_N-X_{N-1})^\nu\\ &+2\mathcal{F}_{\mu\nu}\dot{X}_0^\mu(X_1-X_0)^\nu+2\mathcal{F}_{\mu\nu}\dot{X}_1^\mu(X_2-X_1)^\nu+\cdots\\ &+2\mathcal{F}_{\mu\nu}\dot{X}_{N-1}^\mu(X_{N-1}-X_{N-2})^\nu\\ &+2\mathcal{F}_{\mu\nu}\dot{X}_N^\mu(X_N-X_{N-1})^\nu\Big], \end{aligned}\quad (19)$$

$$\frac{1}{\varepsilon}(X_1-X_0)^\mu-\mathcal{F}^\mu{}_\nu\partial_\tau X_0^\nu=0, \quad (20)$$

$$\frac{1}{\varepsilon}(X_N-X_{N-1})^\mu-\mathcal{F}^\mu{}_\nu\partial_\tau X_N^\mu=0. \quad (21)$$

In fact, (20) and (21) are not only the BCs but also the equations of motion of the end points X_0 and X_N in the discrete form, and the equations of motion of the middle points X_i, $i\neq 0,N$ in the discrete form are

$$\varepsilon\partial_\tau^2 X_i^\mu=\frac{1}{\varepsilon}(X_{i+1}-2X_i+X_{i-1})^\mu, \qquad i\neq 0,N. \quad (22)$$

Now there are two choices to proceed by. One is the traditional Dirac method. It takes the BCs (20) (or (21)) as the Hamiltonian primary constraints [5,9] in which the Lagrange multipliers are introduced in order to construct the so-called total Hamiltonian, and then it exhausts all the constraint chains or determines the Lagrangian multipliers by the consistency requirements; finally the commutation relations can be obtained by calculating the Dirac brackets. However, two new features beyond the standard Dirac context would appear if we treat the BCs as the Dirac primary constraints [5,9]. One is that the Lagrange multipliers are determined by the consistency requirements while the constraint chains are not terminated. The other is that the constraint chains are infinite. It is quite amazing that such situations rarely occurred before, and it is also quite suspect because the end points' phase space would be of a negative infinite dimension. In a recent paper [7], the author finds that in the Dirac context, if the BCs are treated as the primary constraints, then the constraint chains are not infinite but finite, and the non-commutative algebras will not appear; furthermore, this author stresses that the non-commutative algebras will not appear even if one insists that the constraint chains are infinite. So in this paper, we shall analyze this problem by using the FJ method.

According to the FJ [11] method reviewed in the previous section, it is necessary to find the reduced phase space and re-express the action (19) in a first-order form in this reduced phase space. Observing that the BCs (20) and (21) are not very complicated, it is possible to obtain the reduced phase space by solving them. In doing so, we solve the BCs (20) and (21) and substitute them into the Lagrangian (19), the reduced phase space[1] is obtained and

[1] After the first version of this paper was completed, we were informed by Dr. Christian Grosche that the "reduced phase space" has been discussed in [9]; however the meaning for these authors is different from ours

the action can be written as the first-order form,

$$S = \frac{1}{2}\int \mathrm{d}t[(\mathcal{F}^{-1}M)_{\mu\nu}(X_1 - X_0)^\mu \dot{X}_0^\nu + (\mathcal{F}^{-1}M)_{\mu\nu}(X_N - X_{N-1})^\mu \dot{X}_N^\nu + \mathcal{L}_m], \quad (23)$$

where $M = 1 - \mathcal{F}^2$, and $\mathcal{L}_m$ stands for the Lagrangian which contains all the points except the points X_0^μ, X_N^μ,

$$\begin{aligned}\mathcal{L}_m = &-\varepsilon\eta_{\mu\nu}\dot{X}_1^\mu\dot{X}_1^\nu - \cdots - \varepsilon\eta_{\mu\nu}\dot{X}_{N-1}^\mu\dot{X}_{N-1}^\nu \\ &+\frac{1}{\varepsilon}\eta_{\mu\nu}(X_2 - X_1)^\mu(X_2 - X_1)^\nu + \cdots \\ &+\frac{1}{\varepsilon}\eta_{\mu\nu}(X_{N-1} - X_{N-2})^\mu(X_{N-1} - X_{N-2})^\nu \\ &+2\mathcal{F}_{\mu\nu}\dot{X}_1^\mu(X_2 - X_1)^\nu + \cdots \\ &+2\mathcal{F}_{\mu\nu}\dot{X}_{N-1}^\mu(X_{N-1} - X_{N-2})^\nu. \end{aligned} \quad (24)$$

As there are no constraints on the variables $X_1^\mu, X_2^\mu, \cdots, X_{N-1}^\mu$, the Lagrangian $\mathcal{L}_m$ is treated in the standard way, that is, we introduce the conjugate momenta $P_{i\mu}$ to $X_i^\mu (i = 1, 2, \cdots, N-1)$. These are defined as usual:

$$P_{i\mu} = \frac{\delta S}{\delta \dot{X}_i^\mu} = \mathcal{F}_{\mu\nu}(X_{i+1} - X_i)^\nu - \epsilon\dot{X}_{i\mu}, \quad (25)$$

and the Hamiltonian corresponding to $\mathcal{L}_m$ is

$$\begin{aligned}H_m &= P_{1\mu}\dot{X}_1^\mu + P_{2\mu}\dot{X}_2^\mu + \cdots + P_{(N-1)\mu}\dot{X}_{N-1}^\mu - \mathcal{L}_m \\ &= -\frac{1}{2\varepsilon}P_{1\mu}P_1^\mu - \frac{1}{2\epsilon}M_{\mu\nu}(X_2 - X_1)^\mu(X_2 - X_1)^\nu \\ &\quad +\frac{1}{\epsilon}P_1^\mu\mathcal{F}_{\mu\nu}(X_2 - X_1)^\nu \\ &\quad -\frac{1}{2\varepsilon}P_{2\mu}P_2^\mu - \frac{1}{2\epsilon}M_{\mu\nu}(X_3 - X_2)^\mu(X_3 - X_2)^\nu \\ &\quad +\frac{1}{\epsilon}P_2^\mu\mathcal{F}_{\mu\nu}(X_3 - X_2)^\nu \\ &\quad + \cdots + \cdots - \frac{1}{2\varepsilon}P_{(N-1)_\mu}P^\mu_{(N-1)} \\ &\quad -\frac{1}{2\epsilon}M_{\mu\nu}(X_{N-1} - X_{N-2})^\mu(X_{N-1} - X_{N-2})^\nu \\ &\quad +\frac{1}{\epsilon}P_{N-1}^\mu\mathcal{F}_{\mu\nu}(X_{N-1} - X_{N-2})^\nu. \end{aligned} \quad (26)$$

Hence, the Lagrangian $\mathcal{L}_m$ is written in the first-order form as

$$\mathcal{L}_m = P_{i\mu}\dot{X}_i^\mu - H_m, \quad (27)$$

where the Hamiltonian H_m has been given in (26).

We have "translated" the action (19) into the first-order form, which is necessary for the symplectic quantization,

$$\begin{aligned}S = \int \mathrm{d}t\Big\{&\frac{1}{2}(\mathcal{F}^{-1}M)_{\mu\nu}(X_1 - X_0)^\mu\dot{X}_0^\nu \\ &+\frac{1}{2}(\mathcal{F}^{-1}M)_{\mu\nu}(X_N - X_{N-1})^\mu\dot{X}_N^\nu \\ &+P_{1\mu}\dot{X}_1^\mu + P_{2\mu}\dot{X}_2^\mu + \cdots + P_{(N-1)\mu}\dot{X}_{N-1}^\mu - H_m\Big\}. \end{aligned} \quad (28)$$

A set of symplectic variables,

$$\xi_i^\mu = \left(X_0^\mu, X_1^\mu, P_1^\mu, X_2^\mu, P_2^\mu, \cdots, X_{N-1}^\mu, P_{N-1}^\mu, X_N^\mu\right),$$

and the corresponding canonical one-form,

$$\begin{aligned}a_{i\mu} = &\left(\tfrac{1}{2}(\mathcal{F}^{-1}M)_{\nu\mu}(X_1 - X_0)^\nu, P_{1\mu}, 0, P_{2\mu}, 0, \cdots, P_{N-1}^\mu, 0,\right. \\ &\left. -\tfrac{1}{2}(\mathcal{F}^{-1}M)_{\nu\mu}(X_N - X_{N-1})^\nu\right),\end{aligned}$$

can be read from the action (28). These result in the symplectic two-form matrix f. We have

$$(f_{\mu\nu})_{ij} = \frac{\partial(a_\nu)_j}{\partial(\xi^\mu)^i} - \frac{\partial(a_\mu)_i}{\partial(\xi^\nu)^j}. \quad (29)$$

According to FJ, if the matrix f is not degenerate, then the commutators can be read from its inverse directly. To show how the FJ method works, for the sake of simplicity and without loss of generality, we restrict ourselves to the D_2-brane [5], in this case $\mu, \nu = 1, 2$. After some simple calculations, the explicit expression for the matrix f can be obtained. We give the explicit expression of the matrix here:

$$f = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & B \end{pmatrix}, \quad (30)$$

where A and B are 6×6 matrices, and the I stands for the unitary matrix. We give the explicit expression for the matrix A and B as follows:

$$A = \begin{pmatrix} 0 & -\frac{(\mathcal{F}^{-1}M)_{12}}{2\pi\alpha'} & 0 & \frac{(\mathcal{F}^{-1}M)_{12}}{4\pi\alpha'} & 0 & 0 \\ \frac{(\mathcal{F}^{-1}M)_{12}}{2\pi\alpha'} & 0 & -\frac{(\mathcal{F}^{-1}M)_{12}}{4\pi\alpha'} & 0 & 0 & 0 \\ 0 & \frac{(\mathcal{F}^{-1}M)_{12}}{4\pi\alpha'} & 0 & 0 & -1 & 0 \\ -\frac{(\mathcal{F}^{-1}M)_{12}}{4\pi\alpha'} & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad (31)$$

and

$$B = \begin{pmatrix} 0 & \frac{(\mathcal{F}^{-1}M)_{12}}{2\pi\alpha'} & 0 & -\frac{(\mathcal{F}^{-1}M)_{12}}{4\pi\alpha'} & 0 & 0 \\ -\frac{(\mathcal{F}^{-1}M)_{12}}{2\pi\alpha'} & 0 & \frac{(\mathcal{F}^{-1}M)_{12}}{4\pi\alpha'} & 0 & 0 & 0 \\ 0 & -\frac{(\mathcal{F}^{-1}M)_{12}}{4\pi\alpha'} & 0 & 0 & -1 & 0 \\ \frac{(\mathcal{F}^{-1}M)_{12}}{4\pi\alpha'} & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}. \quad (32)$$

Here we recover the coefficient $2\pi\alpha'$ explicitly. Obviously, f is not singular provided $\mathcal{F}_{\mu\nu}$ is non-vanishing, hence the inverse of this matrix exists,

$$f^{-1} = \begin{pmatrix} A^{-1} & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & 0 & B^{-1} \end{pmatrix}, \quad (33)$$

where A^{-1} and B^{-1} are the inverses of A and B, respectively. For future use, we give them explicitly as follows:

$$A^{-1} = \tag{34}$$
$$\begin{pmatrix} 0 & 2\pi\alpha'(M^{-1}\mathcal{F})_{12} & 0 & 0 & \frac{1}{2} & 0 \\ -2\pi\alpha'(M^{-1}\mathcal{F})_{12} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{1}{2} & 0 & -1 & 0 & 0 & \frac{(M^{-1}\mathcal{F})_{12}}{8\pi\alpha'} \\ 0 & -\frac{1}{2} & 0 & -1 & -\frac{(M^{-1}\mathcal{F})_{12}}{8\pi\alpha'} & 0 \end{pmatrix}$$

and

$$B^{-1} = \tag{35}$$
$$\begin{pmatrix} 0 & -2\pi\alpha'(M^{-1}\mathcal{F})_{12} & 0 & 0 & \frac{1}{2} & 0 \\ 2\pi\alpha'(M^{-1}\mathcal{F})_{12} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{1}{2} & 0 & -1 & 0 & 0 & -\frac{(M^{-1}\mathcal{F})_{12}}{8\pi\alpha'} \\ 0 & -\frac{1}{2} & 0 & -1 & \frac{(M^{-1}\mathcal{F})_{12}}{8\pi\alpha'} & 0 \end{pmatrix}.$$

From the above matrix f^{-1}, we can read off the following relevant commutators:

$$\begin{aligned} \{X_0^\mu, \quad X_0^\nu\} &= 2\pi\alpha'(M^{-1}\mathcal{F})^{\mu\nu}, \\ \{X_0^\mu, \quad X_1^\nu\} &= 0, \\ \{X_1^\mu, \quad X_1^\nu\} &= 0, \\ \{X_0^\mu, \quad P_{1\nu}\} &= \frac{1}{2}\delta_\nu^\mu, \\ \{X_1^\mu, \quad P_{1\nu}\} &= \delta_\nu^\mu, \\ \{X_N^\mu, \quad X_N^\nu\} &= -2\pi\alpha'(M^{-1}\mathcal{F})^{\mu\nu}, \\ \{X_N^\mu, \quad X_{N-1}^\nu\} &= 0, \\ \{X_{N-1}^\mu, \quad X_{N-1}^\nu\} &= 0, \\ \{X_N^\mu, \quad P_{(N-1)\nu}\} &= \frac{1}{2}\delta_\nu^\mu, \\ \{X_{N-1}^\mu, \quad P_{(N-1)\nu}\} &= \delta_\nu^\mu. \end{aligned} \tag{36}$$

The commutators for the variables $X_i^\mu, P_{j\mu}$ $(i, j \neq 0, 1, N, N-1)$ are

$$\begin{aligned} \{X_i^\mu, \quad X_j^\nu\} &= 0, \\ \{X_i^\mu, \quad P_{j\nu}\} &= \delta_{ij}\delta_\nu^\mu, \\ \{P_{i\mu}, \quad P_{j\nu}\} &= 0. \end{aligned} \tag{37}$$

Our results mainly agree with that of [5,9], except for a little difference: there, not only the end points, but also the points which are neighboring to the end points (i.e. X_1^μ or X_{N-1}^μ) are non-commutative; these conflict with the widely-held viewpoints that only the end points are non-commutative.

4 Conclusions and remarks

In this paper, we use the symplectic quantization method to the problem, the open string in the constant background B-field. In fact, it is a problem of how to treat the BCs in both mechanics and field theories. Due to these BCs, the systems cannot be quantized directly by the replacement $\{\quad,\quad\}_P \to \frac{1}{\mathrm{i}}[\quad,\quad]$, because the fundamental Poisson brackets $\{Q^i, P_j\} = \delta_j^i$ or $\{\Phi^i(x), \Pi_j(x')\} = \delta_j^i \delta(x - x')$ conflict with the BCs on the boundary generally, so a careful treatment of the BCs is needed. Previous work on this subject has taken the BCs as the primary Dirac constraints, and then the Dirac process was used. However, some confusions mentioned in the previous sections would arise. Contrary to the Dirac method, those ambiguities could be avoided by using the FJ method. Because the BCs are not very complicated, it is easy to solve them and find the reduced phase space, so we are allowed to work in this reduced phase space.

One may think that it is strange that in our final results the commutators of X_0^μ and $P_{1\nu}$ (or X_N^μ and P_{N-1}^ν) do not vanish. In fact, it is a common question one faces in the discretization. It is well known that the symplectic structure cannot be preserved as perfectly as its continuous counterpart in general during the discretization. Recently, some researches have been done toward solving this problem, and one found that it is possible to preserve the symplectic structure as good as its continuous counterpart if the discretization is performed in the varied steps; this problem deserves further discussion.

Acknowledgements. This work is supported by the National Natural Science Foundation of China with grant No 10247009.

References

1. H. Snyder, Phys. Rev. **71**, 38 (1947)
2. N. Seiberg, E. Witten, JHEP **99009**, 032 (1999), hep-th/9908142
3. C.S. Chu, P.M. Ho, Nucl. Phys. B **550**, (1999), hep-th/9812219
4. C.S. Chu, P.M. Ho, Nucl. Phys. B **568**, 447 (2000), hep-th/9906192
5. F. Ardalan, H. Arfaei, M.M. Sheikh-Jabbari, Nucl. Phys. B **576**, 578 (2000), hep-th/9906161
6. P.A.M. Dirac, Lecture Notes on Quantum Mechanics, Yeshiva University, New York (1964)
7. F. Loran, Phys. Lett. B **544**, 199 (2002)
8. W. He, L. Zhao, Phys. Lett. B **532**, 345 (2002), hep-th/0111041
9. M.M. Sheikh-Jabbari, A. Shirzad, Eur. Phys. J. C **19**, 383 (2001)
10. K-I. Tezuka, Eur. Phys. J. C **25**, 465 (2002)
11. L.D. Faddeev, R. Jackiw, Phys. Rev. Lett. **60**, 1692 (1988)
12. N.R.F. Braga, C.F.L. Godinho, Phys. Rev. D **65**, 085030 (2002)